On the approximation of complex geometric domain to be compatible for the implementation of finite difference method

Md. Masum Murshed, Gour Chandra Paul, Md. Rabiul Haque

Abstract— In this study, an algorithm for approximating a complex coastal geometry to be compatible in implementing finite difference method (FDM) is developed. For a prototype example, a matrix with $(1/120)^0$ resolution of grids representing water, lands, islands and their approximated boundaries through a proper stair step is generated from a colour picture for the coastal region of Bangladesh. The Arakawa C-grid system is used here. The area between $15^0 30''$N and 23^0N Latitudes and 85^0E and 95^0E Longitudes is considered as the physical domain. A MATLAB routine is used to construct 960×1201 computational grids from the colour picture of the domain. By representing the grids with suitable notations, a proper 'stair step' algorithm is developed to approximate the coastal and island boundaries to the nearest finite difference grid lines. Available coastal stations are also identified accurately in the obtained approximated domain. Such a type of presentation of coastal geometry is found to incorporate its complexities properly.

Keywords — Colour picture, MATLAB, Stair step representation, Bay of Bengal, Finite difference method

1 Introduction

GRIDS re defined as smaller shapes formed after discretization of a geometric domain. The finite difference grid matrix is the digital representation of the geometric domain in matrix form after discretization and the elements of the matrix are the node points of the region of finite difference grids. In solving shallow water equations in Cartesian coordinates on a geometri domain, it should be rectangular or square. But in the case of irregular geometry its boundaries should be made straight ones by some suitable transformations or the boundaries should be approximated through proper stair steps. It is of interest to note here that the coast of Bangladesh is familiar for its complexity which lies in the northern tip of the Bay of Bengal. The tropical storms associated with surges cause a tremendous loss of lives and properties every year where the complexity has a great contribution (see [3]). Shallowness of water, thickly populated low laying islands of different shapes, high tidal range, fresh water and sediment discharge through the world's second largest river system Ganges-Brahmaputra-Meghna etc. make this region the world's most vulnerable one, but the area is insufficiently studied. Thus the coastal region of Bangladesh is chosen as our study area. Many analyses on prediction of surge due to tropical storms have been made for the Bay of Bengal region covering the coastal area of Bangladesh and the East coast of India implementing FDM, where the coastal curvilinearity was approximated through stair steps. Out of them worth mentioning are [1]–[6]. The pioneering work due to Das [1] considered a uniform grid distance of 13.8 km throughout the model and the geometric domain was discretized manually. Therefore, such a discretization may lead to a misrepresentation of the surge development as the resolution of the model was not high enough and the geometric domain was discretized manually. It is known that the accuracy of results from a stair step model depends among other on grid resolution. Taking this fact into account, in [5] Roy investigated storm surge problem using a one way nested grid technique, where the resolutions of grids of the child model were Δx = 1388.75 m, Δy = 1288.76 m and those for the parent model were Δx = 22.2 km, Δy = 21.0 km. It is pertinent to point out here that only two islands Char Madras (Bhola) and Char Chenga (Hatiya) with proper stair step representation were inserted in that study. But there are many small and big islands in the offshore region of the Bangladesh coast, which may significantly influence surge intensity (see [2]). Hence inclusion of all islands irrespective of small and big in a model is essential in order to accurate prediction of water levels due to surge associated with a storm. Based on that idea, in [3] Paul and Ismail incorporated all offshore islands along the coast of Bangladesh through proper stair step representation of coastal and island boundaries exercising nesting models and found offshore islands to increase water level in the region of interest. But in all the three investigations the geometric domain was discretized manually and an improved grid resolution could be considered for the whole coast instead of Meghna estuarine area. It is to be noted here that the studies mentioned were conducted including only the Meghna estuarine region considering its vulnerability. Since now-a-days the computational capability of available computers are very high, so a more effective storm surge forecasting model can be developed by increasing the resolution of the computational grids and approximating the geometry accurately through a proper stair step representation using the high computational power of computers under a pr-

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ogramming language.

In this study, we intend to represent accurately an irregular geometry (the coast of Bangladesh) digitally with a proper stair step representation through a computer program to be compatible of using finite difference technique which in turns can be used for digital representation of any coastal region.

2 NECESSARY BACKGROUND

2.1 Grid generation

The smaller shapes, formed after discretization of a geometric domain are known as grids. There are three types of grids namely, structured or staggered grid, unstructured grid and hybrid grid. A structured grid is defined by uniform connec-

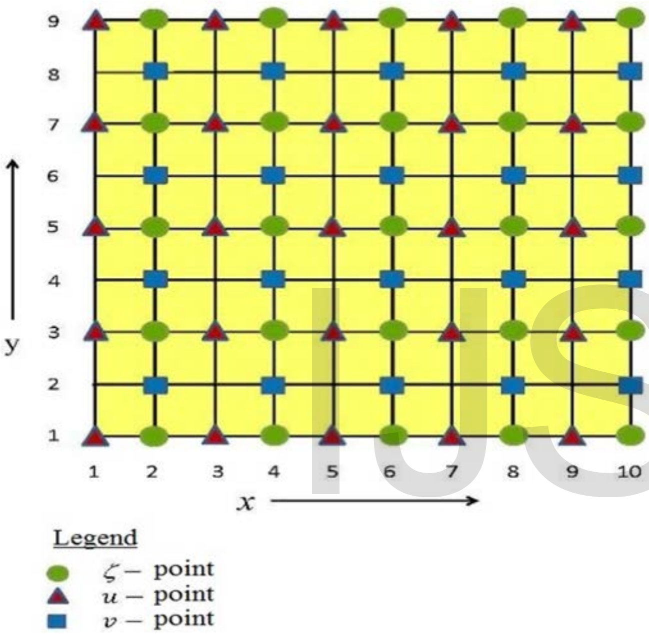

Fig. 1 Arakawa C-grid system used in the study

tivity. Such a type of grid can be expressible as an array of two or three dimension, which restricts the choices of elements to quadrilaterals in 2D or hexahedra in 3D. The regularity of the connectivity allows researchers to conserve space as neighborhood relationships are defined by the storage arrangement. On the other hand, an unstructured grid can be defined as irregular connectivity. Such a grid is not readily expressed as a two or three dimensional array in computer memory. The storage requirements for an unstructured grid in comparison with a staggard grid can be substantially large since the neighborhood connectivity is stored here explicitly. A hybrid grid is that grid which contains both structured portions as well as unstructured portions. In the present study, the structured Arakawa C-grid system is used in which there are three computational grid points refereed to as ζ -point, u-point and v-point (see Fig. 1). First of all, the ζ -point is the grid point (x_i, y_j), where i is even and j is odd. Secondly, the grid point (x_i, y_j), where both i and j are odd, is referred to as u-point. Finally, if both i and j are even, then the grid point (x_i, y_j), is a v-point.

2.2 Stair step representation

Through stair step representation, coastal and island boundaries of a coastal geometry are approximated as either along the nearest odd y-directed grid lines, so that on that part of the boundary, there are only u-points or along the nearest even x-directed grid lines, so that there are only v-points on that part of the boundary. Thus the boundary of the coast is represented by such a stair step that at each segment there exists only that component of the velocity which is normal to the segment. This is done in order to ensure the vanishing of the normal component of the velocity at the boundary in the numerical scheme. This representation is known as stair step representation. A fictitious coast with its stair step representation is

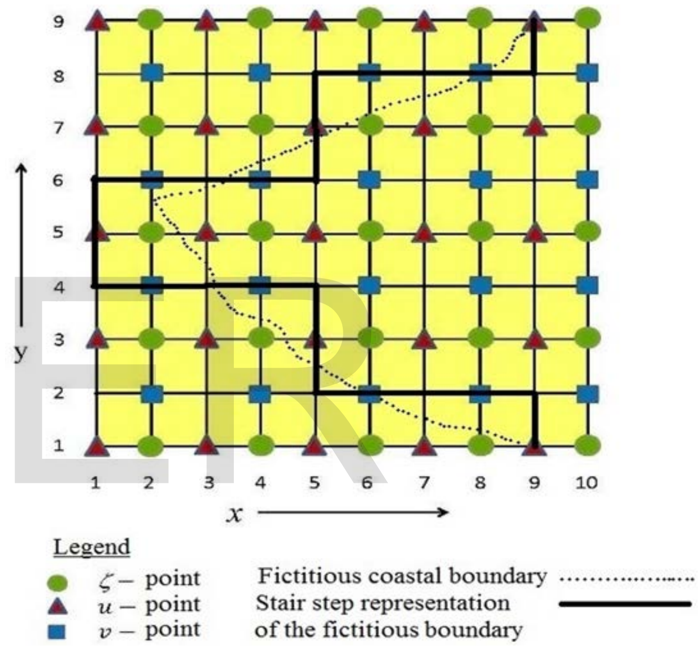

Fig. 2 A fictitious coastal boundary and its stair step representation on the grid system used in this study

shown in Fig. 2.

The 'stair step' representation obtained in the study is on the basis of the following Theorem.

Theorem 1 *If (i, j) is a corner of a stair in the represented 'stair step' form of m × n computational grid matrix with* $1 < j < n$ *then the point (i, j) must be (odd, even).*

Proof A corner (i, j) of the stair in the 'stair step' representation must have any one of the four forms as shown in Fig. 3. It will be shown that (i, j) is (odd, even). It is mentioned earlier that in the 'stair step' representation, the coastal and island boundaries are approximated as either along the nearest odd y-directed grid lines (i.e., i is odd) or along the nearest even x-directed grid lines (i.e., j is even). So that in the y-directed grid lines there are (odd, odd) and (odd, even) points and in the x-

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directed grid lines there are (odd, even) and (even, even) points. Therefore, there does not exist any (even, odd) point in any segment of the stair step representation. Now, suppose that (i, j) is not (odd, even), then (i, j) must be (odd, odd) or (even, even). Now, if (i, j) is (odd, odd), then $(i-1, j)$ and $(i+1, j)$ are (even, odd). Since (i, j) is a corner, either $(i-1, j)$ or $(i+1, j)$ must lie on a segment of the stair step (see Fig. 3). This is a contradiction. Again if (i, j) is (even, even), then $(i, j-1)$ and $(i, j+1)$ are (even, odd), which is also a contradiction. Therefore, (i, j) must be (odd, even).

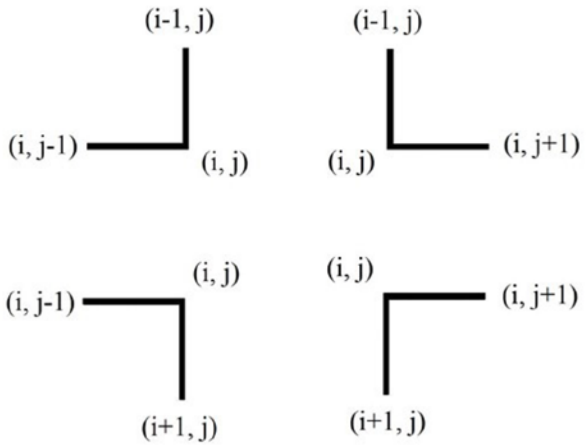

Fig. 3 Positions of corners in different segments found in stair steps

3 METHODOLOGY

Step I

A colour picture of the physical domain lies between Latitudes 15^0N and 23^0N and Longitudes 85^0E and 95^0E (see Fig. 4) is generated through **ArcGIS** software. To employ finite difference technique in solving equations based on a domain of irregular shape it should be approximated with rectangular or square mesh grids, where the curvilinear boundaries are usually approximated through a proper 'stair step' representation.

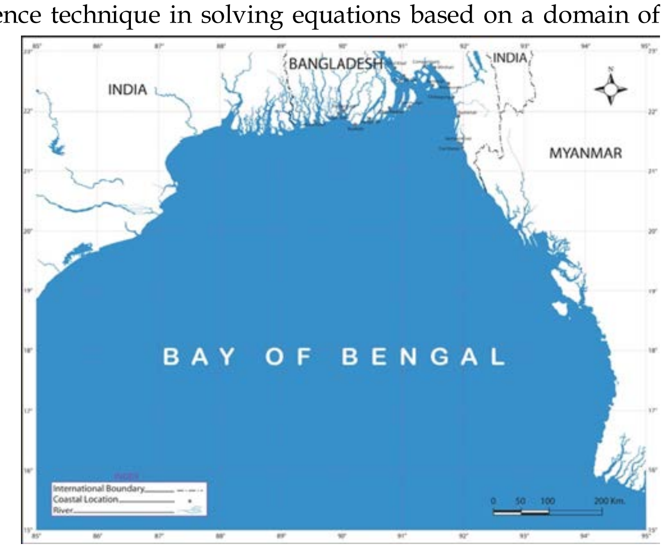

Fig. 4 A colour picture of the northern part of the Bay of Bangle covering the area 15^0-23^0N Latitudes and 85^0-95^0E Longitudes

To make such a representation ensuring $(1/120)^0$ resolution of the grids, the Adobe Illustrator software is firstly used to smooth the map and then Microsoft Office 2010 is used to crop the map having the domain mentioned above. To create $m\times n$ computational grids with uniform grid spacing p and q, respectively, the picture should have the dimension of $((n-1)q+1)\times((m-1)p+1)$ ($width\times height$) pixels. In this study, 960×1201 computational grid points are created with uniform grid spacing $p = q = 6$ so that the dimension of the picture must be of 7201×5755 pixels. Two colour pictures of that dimension are produced by the above process. One of them is with the three colours, namely blue, white and black representing water, land and coastal stations, respectively (see Fig. 5), and the other is with the two colours blue and white representing the same as in the first one.

Fig. 5 Smoothed colour picture of the Bay of Bangle region with 17 stations along the coast of Bangladesh. 1. Hiron point, 2. Tiger point, 3. Patharghata, 4. Kuakata, 5. Rangabali, 6. Char Madras, 7. Charchenga, 8. Chital Khali, 9. Char Jabbar, 10. Companiganj, 11. Mirshari, 12. Shitakunda, 13. Sandwip, 14. Chittagong, 15. Bashkhali, 16. Moheshkhali and 17. Cox's Bazar

Step II

A Matlab routine is used to read the picture with two colours produced in step I and 1201 vatical lines and 960 horizontallines are drawn on it with uniform grid spacing of 6 pixels in both the directions and thus 960×1201 node points of the grids are obtained. Based on the information of colours of the node points of the grids, a matrix of order 960×1201 is generated consisting of 0 and 1 corresponding to white and blue colours, respectively. This matrix is the digital representation of the physical domain, where 1 represents water and 0 represents land of the original domain. Since it is impossible to show the matrix of order 960×1201 in one figure, the approximation process of a portion (see Fig. 6) is shown and it will be continued to show its updated forms in the next subsequent steps. The matrix obtained in this step representing that portion is shown in Fig. 7.

Step III

For the coastal and island boundaries in the original domain, consecutive elements with different digits are produced in the

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matrix obtained in step II. i.e., if the (i, j) th element in the obtained matrix is 1 and (i+1, j) or (i-1, j) or (i, j+1) or (i, j-1) th element is 0, then either 1 or 0 represents the coast of the original domain. The original coastal and island boundaries in the physical domain can be indicated in the obtained matrix by a digit other than 0 and 1. In this step, the coastal and island boundaries are indicated in the obtained matrix by 5 replacing it in the place of 1 in the four different cases mentioned above (see Fig. 8).

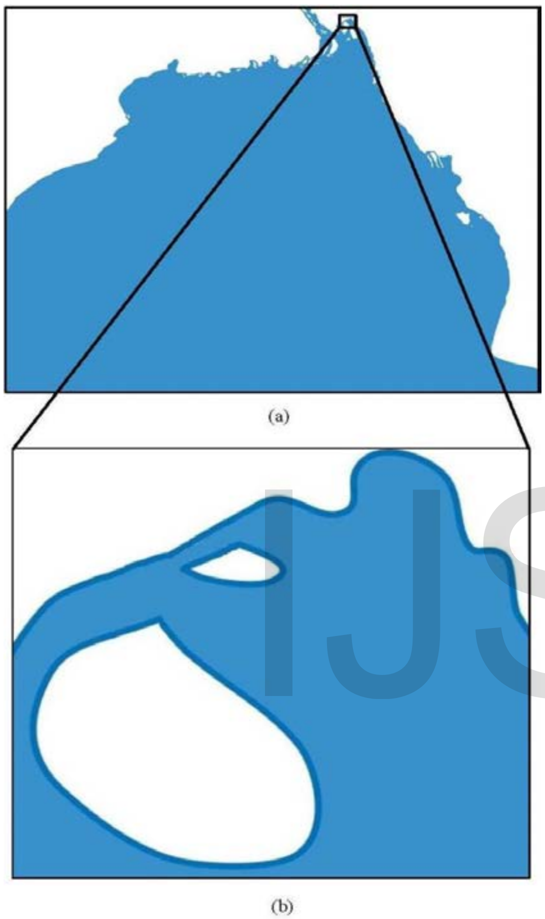

Fig. 6 A portion (b) of the main picture (a) where white colour representsland and blue colour represents water

Step IV

The matrix obtained in step III composed with 0, 1 and 5 represents land, water and the original coastal and island boundaries, respectively. To approximate the coastal and island boundaries through a proper stair step representation, a matrix of order 960×1201 composed with the digits 11, 12, 21 and 22 corresponding to the positions (odd, odd), (odd, even), (even, odd) and (even, even), respectively is generated. The positions (odd, even) and (even, odd) are very important in the stair step representation as the position of a corner of the stair step representation is always (odd, even) (see Theorem 1) and there does not exist any (even, odd) point in any segment of the stair step representation (see subsection 2.2). It may take less time to identify the (i, j) th position to be (odd, even) or (even, odd) from the matrix obtained in this step rather than insuring i and j to be even or odd individually. Thus the matrix constructed in this step will in turn help to reduce the computational costs in the next subsequent steps.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0
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1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 7 The obtained matrix in step II where 0 represents land and 1 represents water

Step V

It is known from the subsection 2.2 that there does not exist any (even, odd) point in any segment of the stair step representation. In this step, the elements near and on the coastal and island boundaries in the obtained matrix other than (even, odd) positions are identified and represented by the digit 4 (see Fig. 9) with the help of the matrix obtained from step IV. For instance, if the (i, j) th element is 5 in the obtained matrix and that is not 21 in the matrix obtained in step IV then 5 is replaced by 4.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1	5	5	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	5	5	5	1	1	1	1	5	1	1	1	1	5	0	0	0	0
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1	5	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	5	0	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1	1
5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1
1	5	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1
1	1	5	0	0	0	0	0	0	0	0	0	0	0	0	5	1	1	1	1	1	1	1	1	1	1
1	1	1	5	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	5	5	5	5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 8 The obtained matrix in step III where 0 represents land, 1 represents water and 5 represents coastal or island boundary

Step VI

The positions of the element 4 in the matrix obtained in step V are (odd, even), (odd, odd) and (even, even). It is known from the Theorem 1 that the position of a corner of the stair step representation is always (odd, even). With the help of the matrix obtained in step IV, the (odd, even) positions are identified in the obtained matrix and represented by 2 based on the logic that if the (i, j) th element is 4 in the updated matrix and that is 21 in the matrix obtained in step IV then 4 is replaced by the digit 2 otherwise (i+1, j) th or (i-1, j) th or (i, j+1) th or (i, j-1) th element is replaced by the digit 2 ensuring that the position in the matrix obtained in step IV is 21 (see Fig. 10).

Step VII

In this step the middle segment of two corners is identified and represented by 2 in the matrix obtained in step VI. For instance, if the (i, j) and (i, j+2) th elements are both 2 then the (i, j+1) th element is replaced by 2 provided that (i, j+1) th element is not 21 in the matrix obtained in step IV and if the (i, j) and (i+2, j) th elements are both 2 then the (i+1, j) th element is replaced by 2 provided that (i+1, j) th element is not 21 in the matrix obtained in step IV. The matrix obtained after this step of that portion is shown in Fig. 11.

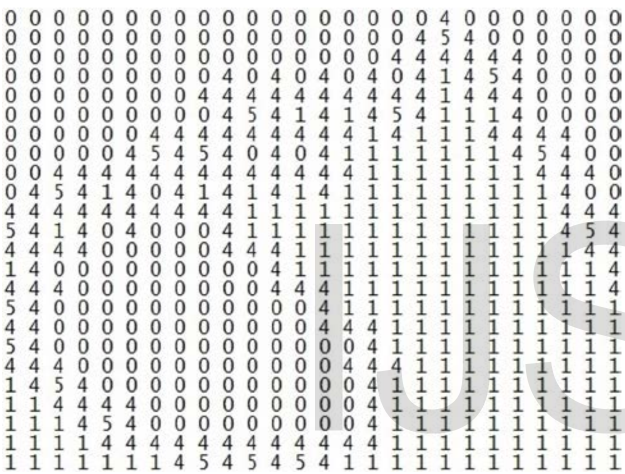

Fig. 9 The obtained matrix in step V with where 0 represents land, 1 represents water, 4 represents water or land near the coast and 5 represents coastal or island boundary

Step VIII

From 11, it is seen that near the coastal and island boundaries some square boxes are created with edges formed by 2 surrounding 0, 1 and 5. In this step, firstly the land area and the water area near the coastal and island boundaries are merged with their own region. The technique for this purpose is that if the (i, j) and (i, j+2) th elements are both 0 (or 1) and the (i, j+1) th element is 2 then the (i, j+1) th element is replaced by 0 (or 1) and if the (i, j) and (i+2, j) th elements are both 0 (or 1) and the (i+1, j) th element is 2 then the (i+1, j) th element is replaced by 0 (or 1) (see Fig. 12). Then square boxes surrounding the digit 5 are merged with the nearest water region or land region in the same procedure. The matrix obtained after this step of that portion is shown in Fig. 13.

Step IX

Some small rivers are disconnected in some places in the matrix obtained in step VIII. As for example, a barrier is indicated by circling 2 (see Fig. 13). In this step, those rivers are connected by the methodology that if the (i, j) and (i+2, j+2) th elements are both 1 and the (i+1, j+1), (i, j+1), (i-1, j+1), (i+2, j+1), (i+3, j+1), (i+1, j+2), (i+1, j+3), (i+1, j), (i+1, j-1) th elements are all 2 then the (i+1, j), (i+2, j) and (i+2, j+1) th elements are replaced by 1 and (i+2, j-1), (i+3, j-1) and (i+3, j) th elements are replaced by 2. A similar work is done for the case where both (i, j) and (i+2, j-2) th elements are 1 and the (i+1, j-1) th element is 2. After this step the stair step representation is completed. The matrix obtained after this step of that portion is shown in Fig. 14.

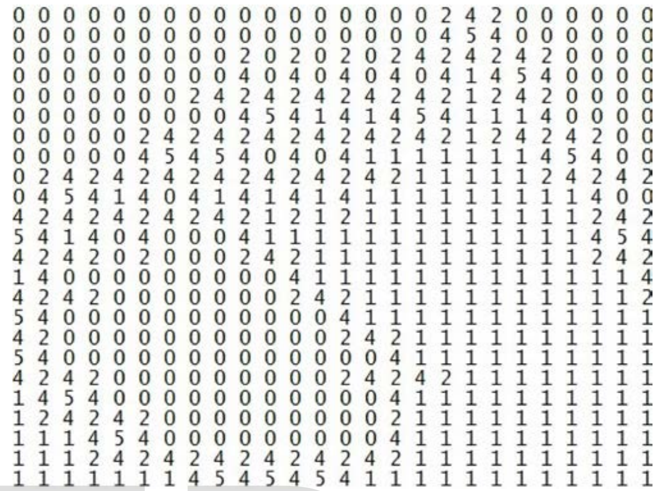

Fig. 10 The obtained matrix in step VI where 0 represents land, 1 represents water, 2 represents corner of the stairs, 4 represents water or land near the coastal and island boundary and 5 represents coastal or island boundary

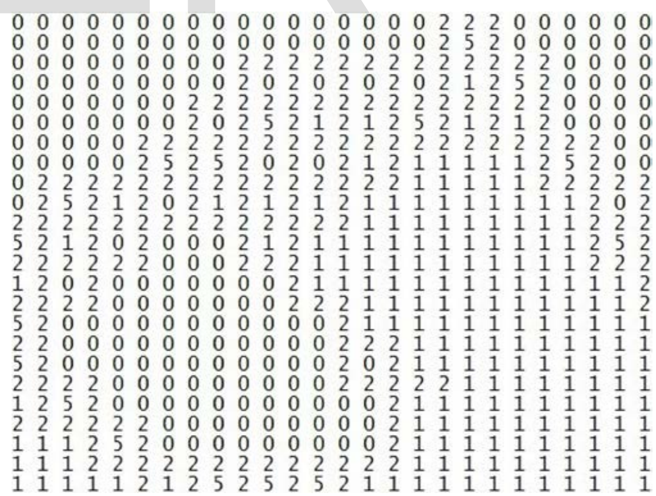

Fig. 11 The obtained matrix in step VII where 0 represents land, 1 represents water, 2 represents approximated coastal and island boundary and 5 represents coastal or island boundary

Step X

Finally, the positions of the coastal stations are identified in the obtained matrix in this step. For this purpose, another matrix of order 960×1201 consisting of 0, 1 and 3 representing land, water and costal stations, respectively, is generated from the picture with three colours obtained in Step 1. Thus the digit 3 is created in the obtained matrix corresponding to the sta-

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tion locations in the picture. Costal stations are assumed to be located in the water. If any (i, j) th element in the matrix obtained in step X is 3 and the corresponding (i, j) th element in the matrix obtained in step IX is 1, then the (i, j) th position is considered as the location of that specific station for which 3 is created, otherwise the nearest position where in the digit 1 exists is considered as the location for that specific station.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	5	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	5	2	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	1	2	2	2	0	0	0	0
0	0	0	0	0	0	0	0	0	2	5	2	1	1	1	2	5	2	1	1	1	2	0	0	0	0
0	0	0	0	0	2	2	2	2	2	2	2	2	2	1	2	2	2	1	1	1	2	2	2	0	0
0	0	0	0	0	2	5	2	5	2	0	0	0	2	1	1	1	1	1	1	1	2	5	2	0	0
0	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	2	2	2	0	0
0	2	5	2	1	2	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	0	0
2	2	2	2	2	2	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
5	2	1	2	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	5	2
2	2	2	2	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	2	2	2
1	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2
2	2	0	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	2
5	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1
2	2	0	0	0	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1
5	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	2	5	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	2	5	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	2	5	2	5	2	5	2	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 12 The obtained matrix in step VIII where 0 represents land, 1 represents water, 2 represents approximated coastal and island boundary and 5 represents coastal or island boundary

4 DISCUSSION OF RESULTS AND CONCLUSION

An accurate digital representation of the geometric domain covering the area $15^0 30''$–23^0N Latitudes and 85^0–95^0E Longitudes is constructed with a $(1/120)^0$ resolution of grids, where the coastal and island boundaries are approximated through a proper stair step to be compatible of using finite difference technique in solving shallow water equations in various purposes. The shapes of the coasts and islands in the digital representation are found to be similar to their shapes in the original colour picture when transformed into picture (see Figs. 15, 16). Costal station locations in the obtained finite difference matrix are found to be in good agreement with their locations in the original colour picture (see Fig. 17). Any geometric domain can be digitalized by the method used in this study provided that water or land as well as coastal stations of the supplied picture of that domain have specific colours. A point to be noted here that, in step III, if the resolution of grids is high enough then the approximation is not affected much whether 1 or 0 is replaced by 5 but if the grid resolution is low then the approximation of small islands or small rivers will be affected much according as only 0 or only 1 is replaced by 5 in the four cases mentioned in step III. A solution to overcome this situation may be, for two cases 0 should be replaced by 5 and for other two cases 1 should be replaced by 5. Since the grid resolution used in the present study is high enough, either 0 or 1 can be replaced by 5. Resolution of grids can be improved whether necessary and a matrix with grids by the technique can be obtained. The obtained approximated geometry along with its digital representation can be used to solve shallow water equations by finite difference method for the prediction of water levels due to surge, tide, or their interaction or any other purposes, on that domain. The process of approximating a physical domain can be used to approximate any complex costal geometry.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	2	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	2	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	1	1	1	2	0	0	0	0
0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	2	0	0	0	0
0	0	0	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	2	2	2	0	0
0	0	0	2	1	1	1	1	1	2	0	0	0	2	1	1	1	1	1	1	1	1	1	2	0	0
0	2	2	2	1	2	2	2	1	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	0	0
0	2	1	1	1	2	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	0	0
2	2	1	2	2	2	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
1	1	1	2	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
1	2	2	2	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2
1	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	2	2	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	2	2	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 13 The obtained matrix in step VIII where 0 represents land, 1 represents water, 2 represents approximated coastal or island boundary

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	2	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	2	0	0	0	0
0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	0	0	0	0
0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	0	0	0	0
0	0	0	2	2	2	2	2	1	2	2	2	2	2	1	1	1	1	1	1	1	2	2	2	0	0
0	0	0	2	1	1	1	1	1	2	0	0	0	2	1	1	1	1	1	1	1	1	1	2	0	0
0	2	2	2	1	1	1	1	1	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	0	0
0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	0	0
2	2	1	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
1	1	1	2	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
1	2	2	2	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2
1	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1	1	1	1
1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	2	2	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	2	2	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	2	0	0	0	0	0	0	0	0	0	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Fig. 14 The obtained matrix in step IX after a proper stair step representation with 0 in land and 1 in water and 2 in coastal or island boundary

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ETHICAL STATEMENT

The authors declare that they have no conflict of interest.

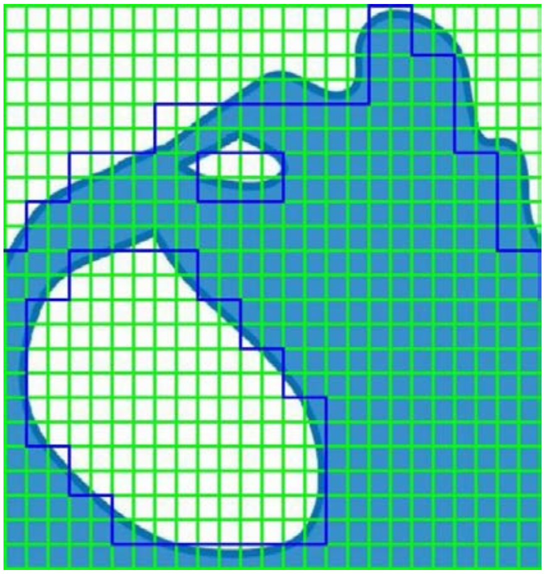

Fig. 15 The representation of land, islands, water and approximated coastal and island boundaries of the portion (b) of the Fig. 6. This representation is obtained after transforming the matrix shown in Fig. 14 into picture

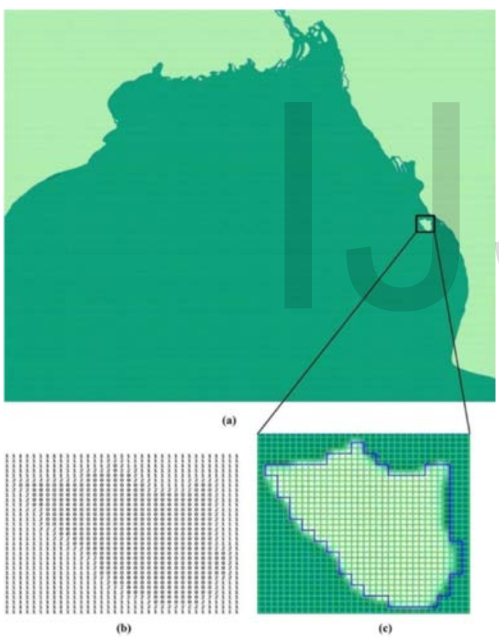

Fig. 16 The representation of another portion of the main picture (a), (b) represents the matrix after completing all the steps mentioned and (c) represents the form when converting (b) into picture

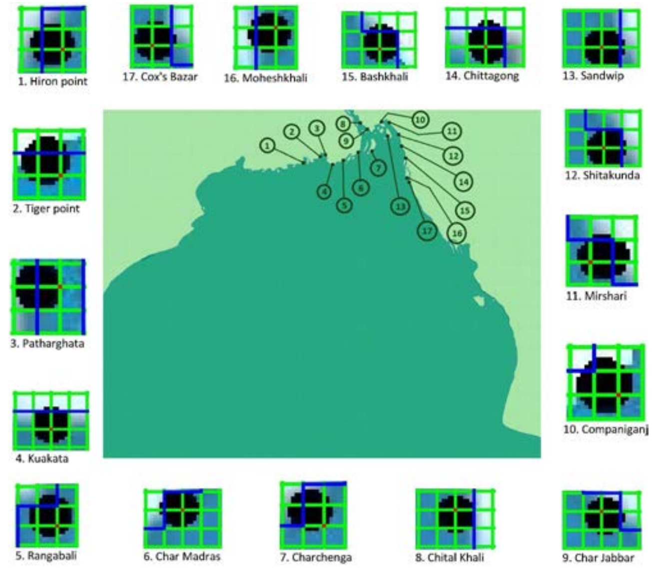

Fig. 17 Approximated costal station locations where red coloured nodes represent the approximated costal stations

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